

Introduction

Metallicity and interstellar absorption are the most important characteristics in studies of globular clusters: the first specifies their basic properties, while the second, on the contrary, makes it difficult to determine them. A number of studies have been devoted to the relationship between parameters of the giants branch (GB) on the color - magnitude diagram and metallicity (for example, Carretta and Bragaglia (1998), Sarajedini and Layden (1997), Zinn and West (1984), Sandage (1982)). However, to determine most of GB parameters, the interstellar absorption should be known. Therefore, it is of great importance to develop a technique for simultaneous determination of both the reddening and the GB parameters. Such techniques were suggested by Sarajedini and Norris (1994) and Ferraro et al. (1999) for $Vvs.(B - V)$, by Sarajedini (1994) for $Vvs.(V - I)$ diagrams. Here, we describe a technique by which the metallicity and the interstellar absorption can be simultaneously derived and which is based on the analysis of $Vvs.(B - R)$ diagrams for globular clusters.

1. The observational data

To develop our technique, a homogeneous set of observations of globular clusters in B, V, R passbands, distributed homogeneously over broad intervals of metallicities and reddenings is needed. We used the B, V, R_c, I_c Johnson-Cousins photometry of stars in globular cluster carried out by Alcaïno et al., which satisfies the above requirements. Table 1 contains the basic parameters of the observed clusters: the name of each cluster, the metallicity in Zinn's scale (see Armandroff and Zinn (1988), Zinn (1985); Zinn and West (1984); this scale, despite some drawbacks, represents the most complete and homogeneous database available), the reddening E_{B-V} and the visual magnitude of horizontal branches (HB) V_{HB} taken from the Harris catalogue (1996, version 2003); the references to published BVR and BV photometrical observations. The BVR magnitudes of stars in three globular clusters (NGC 5286, NGC 6541 and NGC 6723) were kindly presented by N.N.Samus'.

Table 1. The basic parameters of the globular clusters

NGC	$[Fe/H]$	E_{B-V}	V_{HB}	BVR references	BV references
1	2	3	4	5	6
1261	-1.29	0.01	16.7	1	13
1904	-1.68	0.01	16.15	2	14
2298	-1.81	0.14	16.11	3	
2808	-1.71	0.22	16.22	4	15
3201	-1.56	0.23	14.77	5	16
4590	-2.09	0.05	15.68	6	17
5286	-1.79	0.24	16.50	7	
6121	-1.28	0.36	13.45	8	18
6362	-1.08	0.09	15.33	9	19
6541	-1.83	0.14	15.20	10	
6723	-1.09	0.05	15.48	11	
6809	-1.82	0.08	14.4	12	20

Footnote to Table 1: the numbers in columns 5 and 6 denote references to published BVR and BV photometrical observations: 1 - Alcaïno et al. (1992a), 2 - Alcaïno et al. (1994), 3 - Alcaïno and Liller (1986a), Alcaïno et al. (1990a), 4 - Alcaïno et al. (1990b), 5 - Alcaïno et al. (1989), 6 - Alcaïno et al. (1990c), 7 - Samus' et al. (1995), 8 - Alcaïno and Liller (1984), 9 - Alcaïno and Liller (1986b), 10 - Alcaïno et al. (1997), 11 - Alcaïno et al. (1999), 12 - Alcaïno et al.

(1992b), 13 - Ferraro et al. (1993), Alcaino (1979), 14 - Ferraro et al. (1992), 15 - Ferraro et al. (1990), 16 - Lee (1977a), 17 - Walker (1994), 18 - Lee (1977b), 19 - Alcaino et al. (1972), 20 - Lee (1977c)

2. The V vs. $(B - V)$ color-magnitude diagrams.

In order to estimate the quality of B, V, R_c, I_c observations made by Alcaino et al. and their adequacy to our purposes, initially we constructed V vs. $(B - V)$ diagrams for the clusters. For most of the clusters, the bright part of the diagrams appeared to be poorly populated, which is explained by the purpose of the observations: determination of cluster's ages and, hence, obtaining stellar magnitudes for sufficiently faint stars. In turn, it may result in unreliable representation of the GB by some function, which in our case is a cubic polynomial. Therefore, for each cluster from Table 1 we selected the most accurate photometric BV data available in the literature (see references to them in column 6 of Table 1), which represent the total GB. Comparing this set of observations with that obtained by Alcaino et al. and introducing corrections where necessary, we reduced it to the to system of B, V, R, I magnitudes of the latter set.

In the V vs. $(V - B)$ color - magnitude diagrams constructed thereby, the GB for each cluster was represented by a 3-rd order curve, from which the following GB parameters were determined: the GB slope - $S_{2.0}$ (Sandage and Wallerstein, 1960) and $S_{2.5}$ (Sarajedini and Layden, 1997); the height of the GB above the HB level - $\Delta V_{1.1}$, $\Delta V_{1.2}$ (Sarajedini and Layden, 1997), and $\Delta V_{1.4}$ (Sandage and Wallerstein, 1960) in the points with $(B - V)_o = 1.^m1$, $1.^m2$, and $1.^m4$, respectively; the intrinsic $(B - V)_o$ color of the GB $(B - V)_{o,g}$, measured at the HB level. In the calculations for $(B - V)_{o,g}$, $\Delta V_{1.1}$, $\Delta V_{1.2}$, and $\Delta V_{1.4}$, the reddening was taken in accordance with Table 1.

The calculated GB parameters were calibrated with respect to metallicity $[Fe/H]$ with the use of Table 1 data. The correlation (r) of $S_{2.0}$ or $S_{2.5}$ with metallicity is high (see Fig.1):

$$[Fe/H] = 0.64 - 0.34S_{2.0}; r = 0.92; \sigma = 0.14; n = 11, \quad (1)$$

$$[Fe/H] = 0.46 - 0.37S_{2.5}; r = 0.92; \sigma = 0.14; n = 11, \quad (2)$$

where σ is the rms error, n - the number of the clusters. The cluster NGC 6541 which have the largest deviation in fig 1 was excluded from the calculations for the expressions (1), (2) and for the subsequent "metallicity - ΔV " relations.

The correlation between the metallicity and the parameters $(B - V)_{o,g}$, $\Delta V_{1.1}$, $\Delta V_{1.2}$, and $\Delta V_{1.4}$, which depend on interstellar absorption, is slightly worse, within (86 - 89)%. The possible reason for that is some inaccuracy in the adopted values of reddening for the globular clusters that display the largest scattering on calibration curves. Therefore, the values E_{B-V} for these clusters were redefined. Staying true to Sarajedini (1994), we determine the true reddening of a cluster as that which value yields the numerical decision for the system of equations analogous to (3) - (6).

$$[Fe/H] = a1 + b1(B - V)_{o,g} \quad (3)$$

$$[Fe/H] = a2 + b2\Delta V_{1.1} + c2\Delta V_{1.1}^2 \quad (4)$$

$$[Fe/H] = a3 + b3\Delta V_{1.2} + c3\Delta V_{1.2}^2 \quad (5)$$

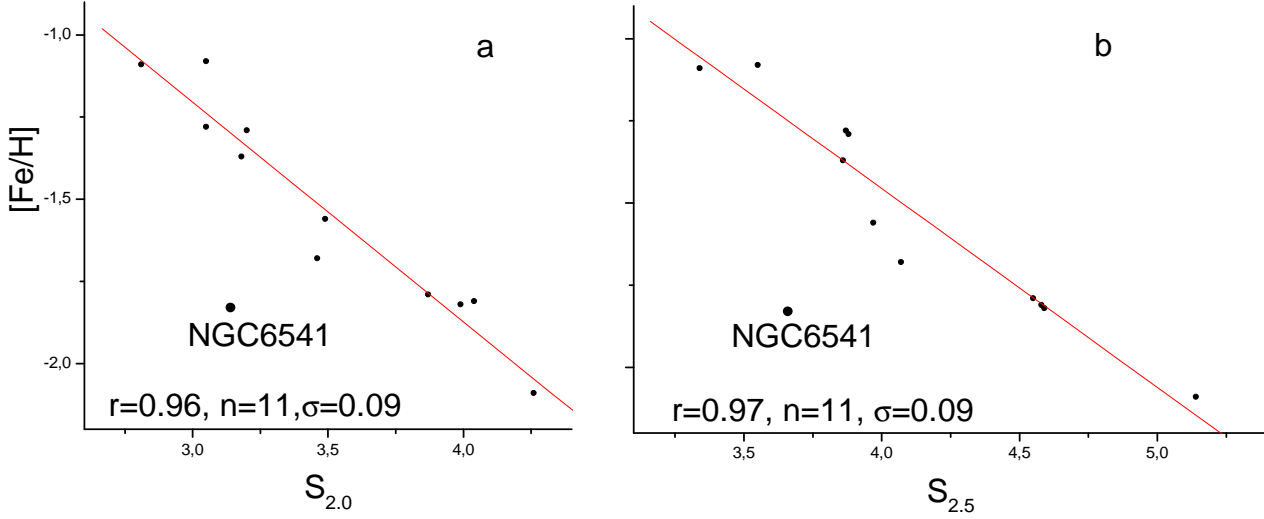


Fig. 1. Calibration of the GB parameters derived from the V vs. $(B - V)$ globular cluster diagrams with respect $[Fe/H]$: (a) - $S_{2.0}$, (b) - $S_{2.5}$. The solid lines are the least-squares fits to the data. The number of the clusters (n) used to compute each relation is reported together with the standard deviations (σ) of the data.

$$[Fe/H] = a_4 + b_4 \Delta V_{1.4} + c_4 \Delta V_{1.4}^2 \quad (6)$$

As a result, for 5 clusters the reddening E_{B-V} is different from that given in Table 1: NGC 2298 - $0.^m21$, NGC 3201 - $0.^m18$, NGC 5286 - $0.^m28$, NGC 6121 - $0.^m40$, NGC 6809 - $0.^m16$. These values, basically, are within the limits in which E_{B-V} are determined by different authors. For example, E_{B-V} received by us exactly coincide with the values obtained by Alcaïno and Liller (1986a) for NGC 2298 and Lee (1977b) for NGC 6121. However, for two clusters the system of equations (3) - (6) has the solution only for following values $E(B - V)$: $E(B - V) \leq 0.^m18$ for NGC 3201 and $E(B - V) \geq 0.^m16$ for NGC 6809. After the acceptance of the new values for color excesses, the system of equations (3) - (6) for clusters from Table 1 (with the exception of NGC 6541 for equations (8) - (10)) has the form (fig 2):

$$[Fe/H] = -5.60 + 5.18(B - V)_{o,g}; r = 0.95; \sigma = 0.10; n = 12, \quad (7)$$

$$[Fe/H] = -1.00 + 0.32\Delta V_{1.1} - 0.28\Delta V_{1.1}^2; r = 0.94; \sigma = 0.09; n = 11, \quad (8)$$

$$[Fe/H] = -0.98 + 0.30\Delta V_{1.2} - 0.22\Delta V_{1.2}^2; r = 0.94; \sigma = 0.09; n = 11, \quad (9)$$

$$[Fe/H] = 1.68 - 1.45\Delta V_{1.4} - 0.11\Delta V_{1.4}^2; r = 0.88; \sigma = 0.10; n = 11 \quad (10)$$

The correlation for $\Delta V_{1.4}$ deteriorates due to the fact that in metal-poor clusters the GB does not reach colors $(B - V)_o \geq 1.4$ and the extrapolation for these colors is unreliable. For metal-rich clusters, this part of the color-magnitude diagram is populated by red variable stars, which also makes the interpolation unreliable.

When the obtained equations are compared with those published in the literature, it is apparent that they agree qualitatively. A direct comparison is possible only with respect to the equation (7). The coefficients of

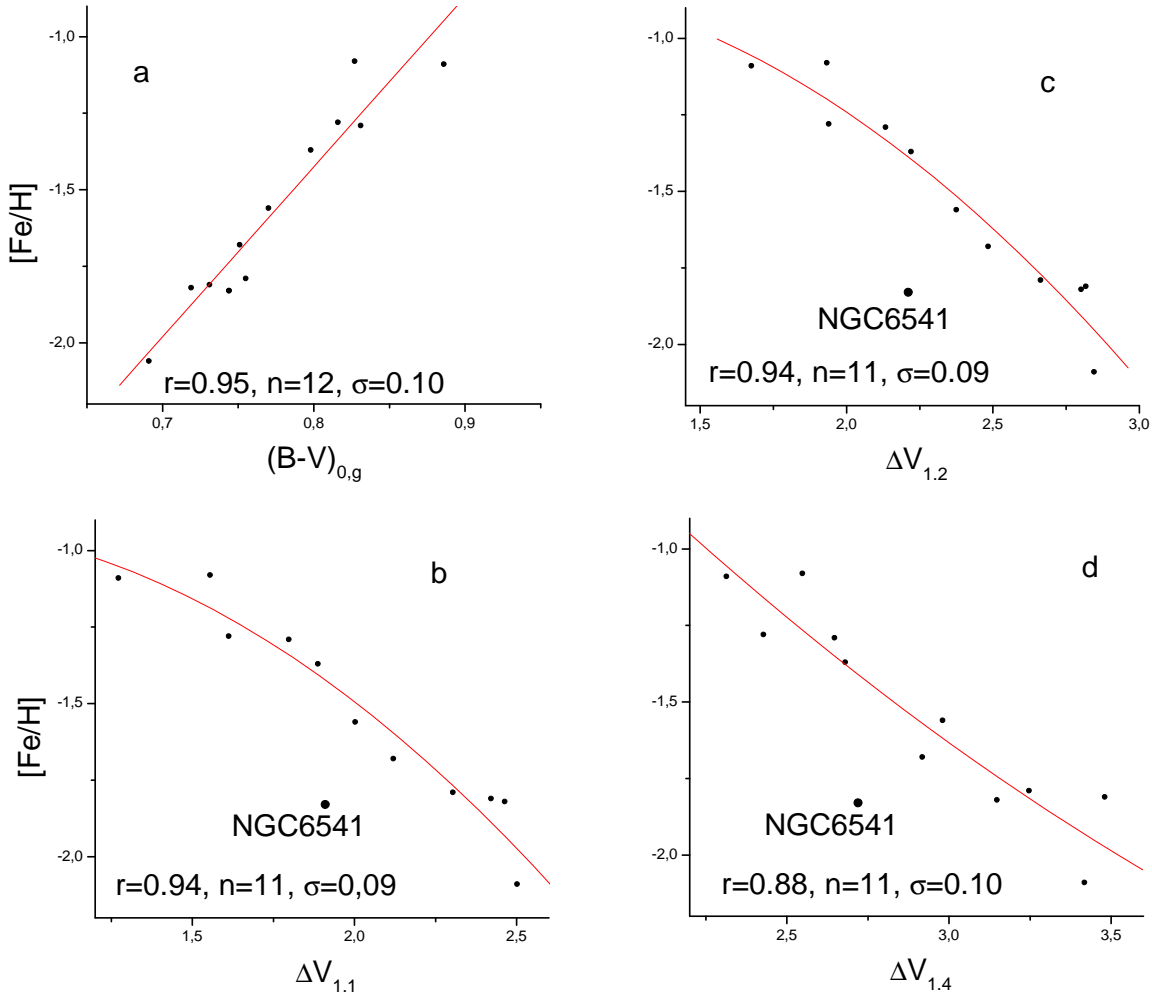


Fig. 2. Calibration of the GB parameters derived from the V vs. $(B - V)$ globular cluster diagrams with respect $[Fe/H]$: a - $(B - V)_{0,g}$, b - $\Delta V_{1,1}$, c - $\Delta V_{1,2}$, d - $\Delta V_{1,4}$. Text is the same as in fig.1.

this equation correspond to those obtained by Sarajedini and Norris (1994) within the errors of determination. For the other equations, no direct comparison is impossible, either because of different scales of metallicity or since the in other studies equations of other degrees are presented.

From the above we can conclude that the observational data obtained by Alcaino with co-authors is consistent with the goals of our study, after the modifications described.

3. The V vs. $(B - R)$ color - magnitude diagrams.

As it was noted above, the data of Alcaino's observations poorly represent the brightest part of the GB on the V vs. $(B - V)$ color - magnitude diagram. It also refers to the V vs. $(B - R)$ or V vs. $(V - R)$ diagrams. Therefore, the construction of these diagrams and determination for the GB parameters on them should be carried out as follows. From the data obtained by Alcaino et al., we derive relations between $(B - V)$ and $(B - R)$, and also between $(B - V)$ and $(V - R)$ colors for each cluster. The correlation of $(B - V)$ and $(B - R)$ colors is high

(> 99%), substantially exceeding that in the case of $(B - V)$ and $(V - R)$ colors, which justifies the selection of the V vs. $(B - R)$ diagram for our study. With the use of these relations, we transform the V vs. $(B - V)$ diagrams described in the previous paragraph into V vs. $(B - R)$. When these diagrams are compared with the data obtained by Alcaino et al., no systematic deviations between them are seen. The GB parameters are determined from V vs. $(B - R)$ diagrams in a way similar to that used for V vs. $(B - V)$ diagrams.

The relations between these parameters and metallicity have been derived using Table 1 and the E_{B-V} values for some clusters, specified in the previous paragraph. Color excesses E_{B-V} are transformed to E_{B-R} through the Grebel and Roberts (1995) ratio: $E_{B-R}/E_{B-V} = 1.62$.

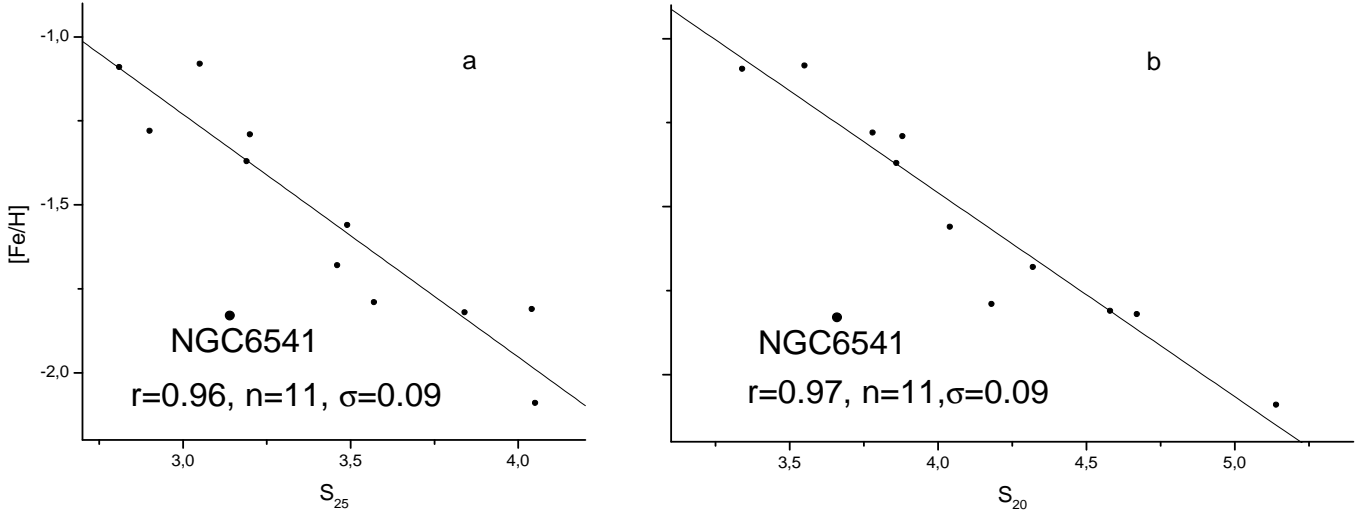


Fig. 3. The same as in fig.1, but for diagrams V vs. $(B - R)$

The equations that connect the GB parameters and metallicity of the clusters (fig 3 and fig 4) can be used to determine the metallicity $[Fe/H]$ and color excesses E_{B-R} :

$$[Fe/H] = 0.97 - 0.61S_{2.0}; r = 0.96; \sigma = 0.09; n = 11, \quad (11)$$

$$[Fe/H] = 0.88 - 0.71S_{2.5}; r = 0.97; \sigma = 0.09; n = 11, \quad (12)$$

$$(B - R)_{o,g} = 1.64 + 0.27[Fe/H]; r = 0.96; \sigma = 0.03; n = 12, \quad (13)$$

$$[Fe/H] = -0.82 + 0.01\Delta V_{1.6} - 0.23\Delta V_{1.6}^2; r = 0.92; \sigma = 0.10; n = 11, \quad (14)$$

$$[Fe/H] = -0.59 + 0.01\Delta V_{1.7} - 0.18\Delta V_{1.7}^2; r = 0.92; \sigma = 0.11; n = 11, \quad (15)$$

$$[Fe/H] = -0.63 + 0.15\Delta V_{1.9} - 0.21\Delta V_{1.9}^2; r = 0.93; \sigma = 0.10; n = 11, \quad (16)$$

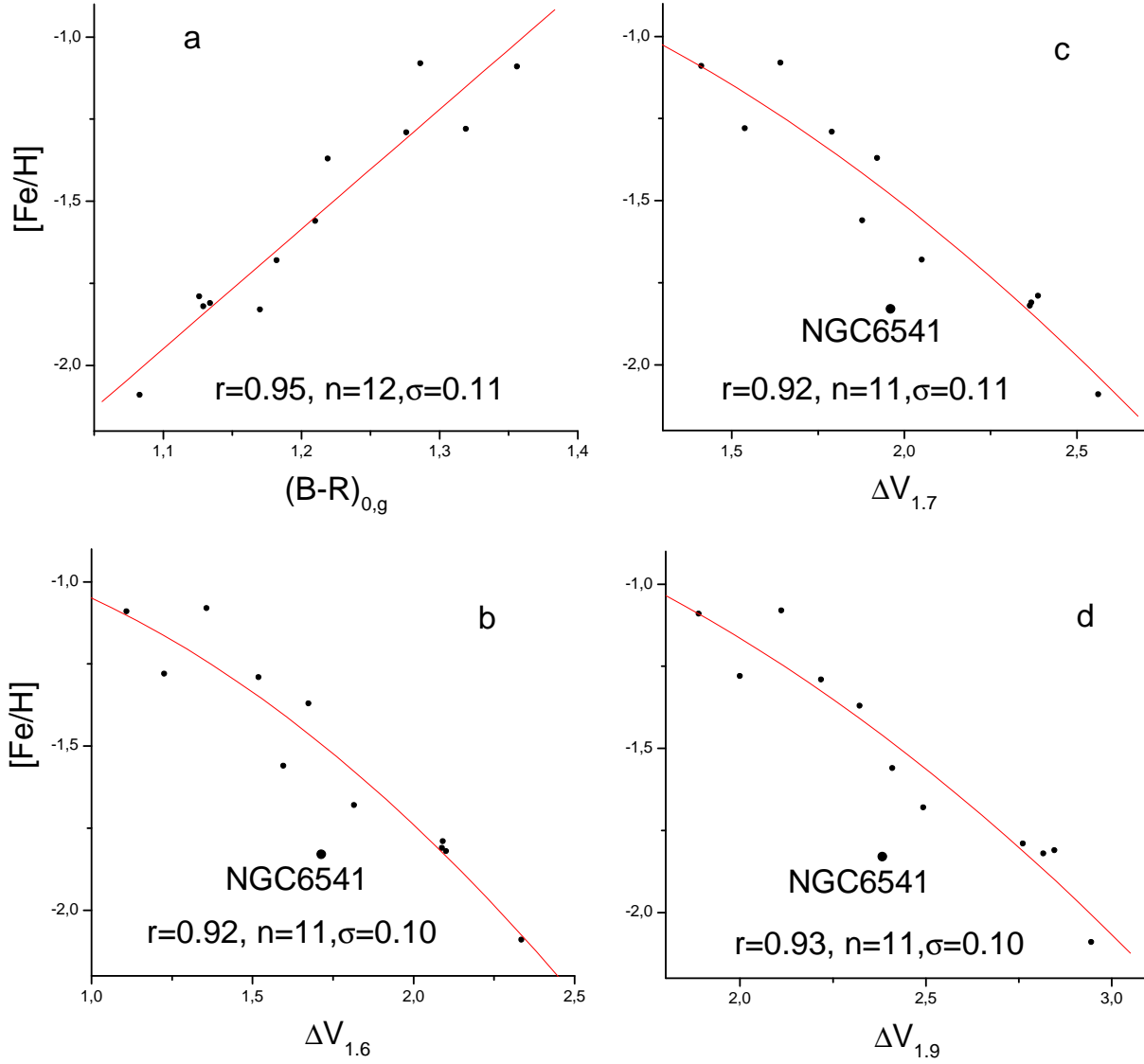


Fig. 4. The same as in fig.2, but for diagrams V vs. $(B - R)$

$$[Fe/H] = -5.96 + 3.64(B - R)_{0,g}; r = 0.95; \sigma = 0.11; n = 12 \quad (17)$$

When the equations (1,2,7-10) are compared with (11-17), their correlation coefficients and rms show that the accuracy of determination of metallicity and color excesses through V vs. $(B - R)$ diagrams is not worse than through V vs. $(B - V)$ diagrams.

Below, we consider the application of the equations (11-17) to simultaneous determination for the metallicity $[Fe/H]$ and color excesses E_{B-R} for the globular cluster NGC 7006.

4. The application of the method to NGC 7006

The observational data for the cluster NGC 7006, used to apply our method, were obtained with the Zeis-1000 Telescope at SAO RAS equipped with the K-585 CCD-detector in B, V, R_c Johnson - Cousins passbands (Gerashchenko, 2007).

NGC 7006 is a cluster of intermediate metallicity; its values, determined by various methods, vary within broad limits (see Table 2). After rejection of several extreme values, the average $[Fe/H] = -1.59$.

Table 2. The determinations of metallicity of NGC 7006

$[Fe/H]$	method	Reference
-2.0	Washington photometry	Canterna (1975)
-1.6	DDO photometry from spectral scans	Hesser et al.(1977)
-1.45	low-resolution spectra	Searle and Zinn (1978)
-1.2	synthetic spectra	Bell and Gustafsson (1978)
-1.53	Q39	Zinn (1980)
-1.59	compilation	Webbink (1981)
-1.9	spectral scans	McClure and Hesser (1981)
-1.45	low-resolution spectra	Friel et al.(1982)
-1.5	low-resolution spectra	Cohen and Frogel (1982)
-1.59	compilation	Zinn and West (1984)
-1.77	$(B - V)_{o,g}$	Buonanno et al.(1991)
-1.63	compilation	Harris (1996)
-1.6	medium-resolution spectra	Wachter et al.(1998)
-1.55	high-resolution spectra	Kraft et al. (1998)
-1.78	V vs. $(B - R)$ diagram	this work

Intense studies of NGC 7006 carried out by Sandage and Wildey (1967) showed that its HB morphology does not correspond to its metallicity. Later on, an entire group of intermediate-metallicity clusters was found, in which HB appears to be redder than it should be for a given metallicity; this discrepancy, known as "the problem of the second parameter", has still not explained.

At the beginning of the 90-ies, in the study of Buonanno et al.(1991) it was assumed that the HB color tends to be bluer with a decrease in the distance from the cluster center. In our study (Gerashchenko, 2007), it was proved for distances from the center of a cluster up to $r \geq 15''$.

In addition to that, in the above study, using the relations obtained in the study of Ferraro et al.(1999) between metallicity and GB parameters derived from the V vs. $(B - V)$ diagram, we determined the metallicity and interstellar absorption for the cluster. The metallicity (-1.62 in the Carretta and Grettton (1997) system) corresponds to the averaged over earlier determinations; however, the color excess ($E_{B-V} = 0.^m15$) exceeds most of previously found values ($E_{B-V} = 0.^m05$).

In our study, we determine the HB parameters from the V vs. $(B - R)$ diagram, from which, using the formula (11 - 17), we calculate metallicity and reddening of the cluster according to two procedures, both including iterative approximations.

One of these procedures was presented in the study by Ferraro et al. (1999), where the initial metallicity is estimated from the formulae (11) and (12) and the interstellar absorption from the ratio $E_{B-R} = (B - R)_g - (B - R)_{o,g}$ and the formulae (13) for $(B - R)_{o,g}$. These values are then improved by iterative calculations

according to the formulae (14-17) until the point when two consecutive estimations for $[Fe/H]$ and E_{B-R} differ less than by 0.1dex, or $0.^m02$, respectively.

The other procedure was suggested by Sarajedini(1994). The system of equations (10-13) is solved iteratively for $E_{B-R} = E_{0,(B-R)} + i\Delta E_{B-R}$, until the point when the difference in metallicity determined from ΔV and $(B-R)_{o,g}$ becomes smaller than 0.1dex. Here, $E_0, B-R$ is the initial value of the color excess E_{B-R} , $\Delta E_{B-R} = 0.1$ is the step of iterations, i - the number of them.

Within the errors of determination, both procedures yield identical results: $E_{B-R} = 0.^m25 \pm 0.^m02$, $[Fe/H] = -1.78 \pm 0.11$; the convergence of the results is reached very rapidly.

5. Conclusion

Here, we have derived relations between GB parameters in the $Vvs.(B-R)$ diagrams for a globular cluster and its metallicity, which make it possible to determine simultaneously the metallicity and interstellar absorption for the cluster by iterative calculations. The accuracy of these determinations appears to be not worse than that reached by a similar method using $Vvs.(B-V)$ diagrams: ± 0.1 dex for the metallicity and $\pm 0.^m02$ for the color excess. The application of this technique to BVR photometry of the cluster NGC 7006 yields $[Fe/H] = -1.78$ and $E_{B-R} = 0.^m25$. The obtained metallicity is consistent with that previously published within the limits of determination errors. The color excess E_{B-R} , after the transformation into E_{B-V} , confirms our previous determination $E_{B-V} = 0.^m15$ (Gerashchenko, 2007)

6. Acknowledgements

The author is grateful to N.N.Samus' for providing photometric data for three clusters, to V.N.Frolov for useful discussion, and to J.K.Ananjevskaya for her assistance in the preparation of this paper.

References

- Alcaino, G.: 1972, *Astron. Astrophys.* 16, 220
- Alcaino, G.: 1979, *Astron. Astrophys. Suppl. Ser.* 38, 61
- Alcaino, G. and Liller, W.: 1984, *Astrophys. J. Suppl. Ser.* 56, 19
- Alcaino, G. and Liller, W.: 1986a, *Astron. Astrophys.* 161, 61
- Alcaino, G. and Liller, W.: 1986b, *Astron. J.* 91, 303
- Alcaino, G., Liller, W. and Alvarado, F.: 1989, *Astron. Astrophys.* 216, 68
- Alcaino, G., Liller, W. and Alvarado, F.: 1990a, *Astron. Astrophys. Suppl. Ser.* 83, 269
- Alcaino, G., Liller, W., Alvarado, F. et al.: 1990b, *Astrophys. J. Suppl. Ser.* 72, 693
- Alcaino, G., Liller, W., Alvarado, F. et al.: 1990c, *Astron. J.* 99, 1831
- Alcaino, G., Liller, W., Alvarado, F. et al.: 1992a, *Astron. J.* 104, 1850
- Alcaino, G., Liller, W., Alvarado, F. et al.: 1992b, *Astron. J.* 104, 190
- Alcaino, G., Liller, W., Alvarado, F. et al.: 1994, *Astron. J.* 107, 230
- Alcaino, G., Liller, W., Alvarado, F. et al.: 1997, *Astron. J.* 114, 2638
- Alcaino, G., Liller, W., Alvarado, F. et al.: 1999 *Astron. Astrophys. Suppl. Ser.* 136, 461
- Armandroff, T.E. and Zinn, R.: 1988, *Astron. J.* 96, 92
- Bell, R.A. and Gustafsson, B.: 1978, *Astron. Astrophys. Suppl. Ser.*, 34, 229
- Buonanno, R., Fusi Pecci, F., Cappellaro, E. et al.: 1991, *Astron. J.* 102, 1005
- Canterna, R.: 1975, *Astrophys. J.* 200, L63
- Carretta, E. and Bragaglia, A.: 1998, *Astron. Astrophys.* 329, 937
- Carretta, E. and Gratton, R.G.: 1997, *Astron. Astrophys. Suppl. Ser.* 121, 95
- Cohen, J.G. and Frogel, J.A.: 1982, *Astrophys. J.* 255, L39

- Ferraro, F.R., Clementini, G., Fusi Pecci F. et al.: 1990, *Astron. Astrophys. Suppl. Ser.* 84, 59
- Ferraro, F.R., Clementini, G., Fusi Pecci F. et al.: 1992, *MNRAS*, 256, 391
- Ferraro, F.R., Clementini, G., Fusi Pecci F. et al.: 1993, *MNRAS*, 264, 273
- Ferraro, F.R., Messineo, M., Fusi Pecci F. et al.: 1999, *Astron. J.* 118, 1738
- Friel, E., Kraft, R.P., Suntzeff, N.B. et al.: 1982, *Publ. Astron. Soc. Pacific* 94, 873
- Gerashchenko, A.N.: 2007, *A.Rep.* 51, 542
- Grebel, A.K. and Roberts, W.J.: 1995, *Astron. Astrophys. Suppl. Ser.*, 109, 293
- Harris, W.E.: 1996, *Astron. J.* 112, 1487
- Hesser, J.E., Hartwick, F.D.A. and McClure R.D.: 1977, *Astrophys. J. Suppl. Ser.* 33, 471
- Kraft, R.P., Sneden, C., Smith G.H. et al.: 1998, *Astron. J.* 115, 1500
- Lee, S.-W.: 1977a, *Astron. Astrophys. Suppl. Ser.* 28, 409
- Lee, S.-W.: 1977b, *Astron. Astrophys. Suppl. Ser.* 27, 367
- Lee, S.-W.: 1977c, *Astron. Astrophys. Suppl. Ser.* 29, 1
- McClure, R.D. and Hesser, J.E.: 1981, *Astrophys. J.* 246, 136
- Samus', N., Ipatov, A., Smirnov, O. et al.: 1995, *Astron. Astrophys. Suppl. Ser.* 112, 439
- Sandage, A.: 1982, *Astrophys. J.* 252, 553
- Sandage, A. and Wildey, L.: 1967, *Astrophys. J.* 150, 469
- Sandage, A. and Wallerstein, G.: 1960, *Astrophys. J.* 131, 598
- Sarajedini, A.: 1994, *Astron.J.* 107, 618
- Sarajedini, A. and Layden, A.: 1997, *Astron.J.* 113, 264
- Sarajedini, A. and Norris, L.E.: 1994, *Astrophys. J. Suppl. Ser.* 93, 161
- Searle, L. and Zinn, R.: 1978, *Astrophys. J.* 225, 357
- Zinn, R.J.: 1985, *Astrophys. J.* 293, 424
- Zinn, R.J. and West, M.J.: 1984, *Astrophys. J. Suppl. Ser.* 55, 45
- Wachter, S., Wallerstein, G., Brown, J.A. et al.: 1998, *Publ. Astron. Soc. Pacific* 110, 821
- Walker, A.R.: 1994, *Astron. J.* 108, 555
- Webbink, R.F.: 1985, *Dynamic of star Clusters*, IAU Symposium 113, (Ed. Goodman J. and P. Hut, Reidel, Dordrecht), p. 541